Multiplication of two matrices

Let $A_{m\times n}$, $B_{K\times L}$ be two matrices.

$n = K$ ✓

$A_{m\times n} \cdot B_{K\times L} = C_{m\times L}$

$$AB = D = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1r} \\ a_{21} & a_{22} & \cdots & a_{2r} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ir} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mr} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{r1} & b_{r2} & \cdots & b_{rj} & \cdots & b_{rn} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & d_{ij} & \vdots \\ d_{i1} & d_{i2} & \cdots & d_{in} \\ \vdots & \vdots & & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}$$

$$d_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{ir}b_{rj}$$

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$\underset{2\times 3}{A} \cdot \underset{3\times 4}{B} = \begin{bmatrix} \underset{(1,1)}{12} & \underset{(1,2)}{27} & \underset{(1,3)}{30} & \underset{(1,4)}{13} \\ \underset{(2,1)}{8} & \underset{(2,2)}{-4} & \underset{(2,3)}{26} & \underset{(2,4)}{12} \end{bmatrix}_{2\times 4}$$

$1\cdot 4 + 2\cdot 0 + 4\cdot 2 = 12$

$1\cdot 1 + 2\cdot -1 + 4\cdot 7 = 27$

$1\cdot 4 + 2\cdot 3 + 4\cdot 5 = 30$

$1\cdot 3 + 2\cdot 1 + 4\cdot 2 = 13$

$2\cdot 4 + 6\cdot 0 + 0\cdot 2 = 8$

$2\cdot 1 + 6\cdot -1 + 0\cdot 7 = -4$

$2\cdot 4 + 6\cdot 3 + 0\cdot 5$

$2\cdot 3 + 6\cdot 1 + 0\cdot 2 = 12$

$\underset{3\times 4}{B} \cdot \underset{2\times 3}{A}$ = does not exist. $4 \neq 2$

$AB \neq BA$

Determine whether a product is defined

Matrices A: 3×4, B: 4×7, C: 7×3

Solution

AB defined — $A_{3\times 4} B_{4\times 7} = C_{3\times 7}$

AC not defined — $A_{3\times 4} C_{7\times 3}$ undefined, does not exist

CA defined — $C_{7\times 3} A_{3\times 4} = D_{7\times 4}$

BC defined — $B_{4\times 7} C_{7\times 3} = D_{4\times 3}$

CB not defined — $C_{7\times 3} B_{4\times 7}$ undefined

Conclusion: The product of matrices is ...

$\underset{7\times 4}{(B^T)} \underset{4\times 3}{(A^T)} = D_{7\times 3}$

$$A = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$\underset{1\times 3}{A} \cdot \underset{3\times 4}{B} = \begin{bmatrix} \underset{(1,1)}{12} & \underset{(1,2)}{27} & \underset{(1,3)}{30} & \underset{(1,4)}{13} \end{bmatrix}$$

$1\cdot 4 + 2\cdot 0 + 4\cdot 2 = 12$

$1\cdot 1 + 2\cdot -1 + 4\cdot 7 = 27$

$1\cdot 4 + 2\cdot 3 + 4\cdot 5 = 30$

$1\cdot 3 + 2\cdot 1 + 4\cdot 2 = 13$

$\underset{3\times 4}{B} \cdot \underset{1\times 3}{A}$ undefined

---

$1\cdot 1 + 2\cdot -1 + 4\cdot 7 = 27$

$1\cdot 4 + 2\cdot 3 + 4\cdot 5 = 30$

Let $\underset{1\times 3}{A = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}}$, $\underset{3\times 4}{B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}}$ and $D = A.B$. Then $d_{12} + d_{13}$ is

$$D = \begin{bmatrix} \underset{(1,1)}{\times} & \underset{(1,2)}{27} & \underset{(1,3)}{30} & \underset{(1,4)}{\times} \end{bmatrix}$$

a) 37
b) 47
c) 57 — $30 + 27 = 57$
d) 27
e) −77

Policies

Suppose that A, B, C, D, and E are matrices with the following sizes:

A	B	C	D	E
(3 × 2)	(3 × 2)	(2 × 3)	(3 × 3)	(2 × 3)

$(B^T)_{2\times 3} \cdot D_{3\times 3}$

$E_{2\times 3} \cdot D_{3\times 3}$

Determine whether the matrix expression $B^TD + ED$ is defined.

Matrix expression is defined.

$2\times 3 + 2\times 3 = 2\times 3$

Enter the size of the resulting matrix (enter 'NA' in each box if undefined).

$B^TD + ED$ is a (2 × 3)

Using the matrices

$$A = \begin{bmatrix} 1 & 1 \\ 6 & 7 \\ 4 & 7 \end{bmatrix}_{3\times 2}, B = \begin{bmatrix} 7 & 3 \\ 5 & 7 \end{bmatrix}_{2\times 2} = 3\times 2$$

compute the following.

AB

$$AB = \begin{bmatrix} 12 & 10 \\ 77 & 67 \\ 63 & 61 \end{bmatrix}$$

Chapter 1, Section 1.3, Question 05j

Policies

Using the matrices

$$D = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 3 & -1 \\ 1 & -3 & 1 \end{bmatrix}, E = \begin{bmatrix} -1 & -1 & 3 \\ 3 & 2 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

compute the following.

$\operatorname{tr}(6E^T - D) =$ 17

$\operatorname{tr}(6E^T - D)$

$$6 \cdot \begin{bmatrix} -1 & 3 & -3 \\ -1 & 2 & 1 \\ 3 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 1 \\ 2 & 3 & -1 \\ 1 & -3 & 1 \end{bmatrix} = \begin{bmatrix} -9 & \cdot & \cdot \\ \cdot & 9 & \cdot \\ \cdot & \cdot & 17 \end{bmatrix}$$

Trace $= -9 + 9 + 17$
$= 17$

Find all values of ~~k, if any, that satisfy the equation~~. Enter the exact answer(s).

$A \cdot B \cdot C = A \cdot (BC) = (AB) \cdot C$

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\underset{1\times 3}{\begin{bmatrix} 2 & 2 & k \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}} = \begin{bmatrix} 8 & 6+2k & 4-k \end{bmatrix}_{1\times 3}$$

$$\underset{1\times 3}{\begin{bmatrix} 8 & 6+2k & 4-k \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix}} = 16 + 12 + 4k + 4k - k^2 = 0$$

$28 + 8k - k^2 = 0$

$k = 4 \mp 2\sqrt{11}$

1) Let $K = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 6 \\ 3 & -2 & 4 \end{bmatrix}$. Then calculate trace(2K) + trace(- K) ?

A) 2
B) 5 ✓
C) 7
D) Can not be calculated
E) 3

$2+0+8 = 10$ $\quad + \quad$ $-1+0+-4 = -5$ $\quad = 5$

$2K = \begin{bmatrix} 2 & x & x \\ x & 0 & 6 \\ x & x & 8 \end{bmatrix}, \quad -K = \begin{bmatrix} -1 & x & x \\ x & 0 & x \\ x & x & -4 \end{bmatrix}$

---

2) Which one is **false** ? 2

A) Multiplyig a row by -5 is a row operation ✓
B) A linear system may have only trivial (zero) solution ✓
C) Not every square matrix has an inverse ✓
D) Dividing a row by 4 is a row operation ✓
E) If a matrix is in row echelon form, it must be in reduced row echelon form ✗

b) $\begin{cases} x+y=0 \\ x-y=0 \end{cases} \Rightarrow 2x=0 \Rightarrow x=0 \Rightarrow y=0$

$Ax=0$ has only zero solution (a unique)
if A is invertible

---

echelon form

3) What can we say about the solution to the following system ?

$$\begin{cases} x + y + z = 0 \\ x - y - z = 2 \\ x + y + 2z = 0 \end{cases}$$

~~A) One zero solution and one nonzero solution~~
~~B) Only zero solution~~
~~C) Infinitely many solutions~~
D) A unique nonzero solution
~~E) No solution~~

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & -1 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -2 & -2 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\rightarrow x+y+z=0 \Rightarrow x=1$
$\rightarrow y+z=-1 \Rightarrow y=-1$
$\rightarrow 0x+0y+1z=0$
$\boxed{z=0}$

Linear System

infinetly many

no solution

a unique

zero solution
$x=y=z=0$

non-zero solution
$x=0 \quad x=0$
$y=1 \quad y=0$
$z=2 \quad z=-1$

---

4) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 7 & 3 \\ 2 & 4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ -5 \\ 6 \end{bmatrix}_{3\times1}$. Then which one is **true** ?

~~A) (2, 1) entry in AB is - 24~~
~~B) B is a 1 x 3 matrix~~
~~C) AB is a 1 x 3 matrix~~
~~D) BA is a 3 x 1 matrix~~
E) (1, 2) entry in $B^T A$ is - 11

$A_{3\times3} B_{3\times1} = (AB)_{3\times1}$
$B_{3\times1} A_{3\times3}$ = does not exist

$AB = \begin{bmatrix} \cdot_{1,1} \\ \cdot_{2,1} \\ \cdot_{3,1} \end{bmatrix}$

$-1\cdot0 + 7\cdot-5 + 3\cdot6$
$0+-35+18 = -17$

$(B^T)_{1\times3} A_{3\times3} = \begin{bmatrix} \cdot & \cdot_{1,2} & \cdot \end{bmatrix}$

$\begin{bmatrix} 0 & -5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -1 & 7 & 3 \\ 2 & 4 & -2 \end{bmatrix}$

$0\cdot2 + -5\cdot7 + 6\cdot4$
$0+-35+24 = -11$

---

5) Let A be 5 x 7, B be 7 x 4, C be 7 x 4, D be 3 x 4 matrices. Then which one of the following operations is **defined**: (I is 5 x 5 identity matrix) 5)

~~A) 5C B~~ $C_{7\times4} B_{7\times4}$
~~B) 3DC~~ $D_{3\times4} C_{7\times4}$
C) $6ABC^T$, $A_{5\times7} B_{7\times4} C^T_{4\times7}$
~~D) 4A + 2B + 3D~~
~~E) B + C + I~~
$7\times4, \; 5\times5$

---

6) Let $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$. Then (3,1) entry in A^{-1} is

A) 0
B) $\frac{1}{2}$
C) A is singular
D) 2
E) -1

A: cofactor / along row: $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$
$2\cdot7 + 6\cdot-2 = 2 \neq 0$

$\left[\begin{array}{ccc|ccc} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1/2 & 3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3.5 & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$

---

7) What condition, if any, must a, b, and c satisfy for the following linear system to be **consistent** ? 7)

$$\begin{cases} x_1 + 3x_2 + x_3 = a \\ -x_1 - 2x_2 + x_3 = b \\ 3x_1 + 7x_2 - x_3 = c \end{cases}$$

A) a, b, c are any real numbers.
B) a - 2b - c = 0
C) 2a - 3b + 4c = 0
D) 3a - 2b + c = 0
E) a + 3b - c = 0

$\begin{bmatrix} 1 & 3 & 1 & a \\ -1 & -2 & 1 & b \\ 3 & 7 & -1 & c \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 & a \\ 0 & 1 & 2 & a+b \\ 0 & -2 & -4 & -3a+c \end{bmatrix}$

$-2y+2z = a+b$
$-2y-4z = -2a-2b$ $\quad$ $-2y-4z = -3a+c$

$-2a-2b = -3a+c$
$-2a-2b+3a-c=0$
$a-2b-c=0$

---

8) What can we say about the solution to the following system ?

$$\begin{cases} x + y + z = 1 \\ 3x - y + 6z = 7 \\ 6x + 2y + 9z = 10 \end{cases}$$

(ex.)

A) Exactly three solutions
B) Infinitely many solutions
C) Only trivial (zero) solution
D) A unique nonzero solution
E) No solution

9) Let A and B be 7 x 7 invertible matrices. Which one is **false**. 9) _

A) $(AB)^{-1} = B^{-1} A^{-1}$ ✓
B) $B^0 = I$, where I is 7 x 7 identity matrix ✓
C) $(A^2)^{-1} = (A^{-1})^2$ ✓
D) $(5A)^{-1} = \frac{1}{5} A^{-1}$ ✓
E) The reduced row echelon form of the matrix BA can't be identity matrix. ✗

invertible

---

e. 30

$\det(C^{-1}) = \frac{1}{\det(C)}$ $\quad$ $\det D = \det D^T$

____ 5. If A,B,C, and D are 4×4 matrices, $\det(A)=1, \det(B)=-2, \det(D)=\frac{1}{8}, \det(2ABC^{-1}D^T)=2$. Then $\det(C^2)=?$

$-2\cdot-2$

a. 4
b. 8
c. 1
d. $-\frac{1}{4}$
e. 2

$\det(2ABC^{-1}D^T) = 2$ $\quad$ 4×4

$2^4 \cdot 1 \cdot -2 \cdot \det(C^{-1}) \cdot \frac{1}{8} = 2$

$-32 \cdot \frac{1}{8} \cdot \det(C^{-1}) = 2$

$-4\det(C^{-1}) = 2$
$\det(C^{-1}) = -1/2 \Rightarrow \det(C) = -2$

___ 6. Let A be a 3 x 3 matrix and det (A) = 2. Then how many of the following statements are true?

* $\det(A^T) = -2$ ✗
* $\det(A^{-1}) = \frac{1}{2}$ ✓
* $\det(-A) = -2$ ✓ → $\det(-1 \cdot A) = (-1)^3 \cdot \det A = -1 \cdot 2 = -2$
* $\det(A^2) = 4$ ✓ $2 \cdot 2 = 4$
* $\det(2A) = 4$ ✗ → $2^3 \cdot 2 = 16$

a. 0
b. 1
c. 2
d. 3
e. 5

___ 10. If $A = \begin{bmatrix} a & b & 2 \\ a & b & 0 \\ 2a & 2b & 4 \end{bmatrix}$, *then* $\det(A) = ?$

a. $8ab$
b. $4ab - 4a$
c. 0
d. 4ba
e. It cannot be determined

Cofactor exp. along row 2

$\cancel{a_{21}C_{21}} + \cancel{a_{22}C_{22}} + \cancel{a_{23}C_{23}}$ $= 0$

$C_{21} = (-1)^3 \cdot (4b - 4b) = 0$

$C_{22} = 0$

OR

Since $R3 = 2R1$, $\det = 0$ ✓

1) Let u = (0, -2, 3, 5, 2), v = (4, 3, 1, -1, -2) and w = (2, 2, -2). Then $-\cancel{12}\dfrac{(u \cdot v)\,(v - u)}{\underbrace{\|w\|^2}_{\cancel{12}}}$ is equal to, where . is the dot product

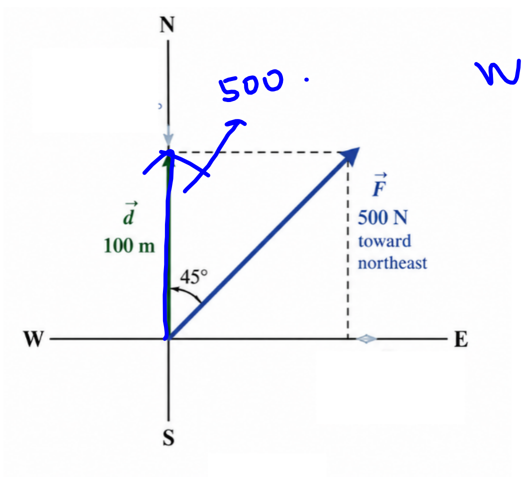
A) (-4, -5, 2, 6, 4)
B) (-4, -5, 2, 3, 3)
C) (0, 0, 0, 0, 0)
D) (-8, -10, 4, 12, 4)
E) Does not make sense (can not be calculated)

$u \cdot v = 0 + -6 + 3 + -5 + -4$
$= -12$.
$\|w\| = \sqrt{4+4+4} = \sqrt{12}$

$-(v-u) = \underline{u - v} = (-4, -5, 2, 6, 4)$

---

2) A sailboat travels 100 m due north while the wind exerts a force of 500 N toward the northeast. How much work does the wind do? Note that Cos 45 = Sin 45 = $\frac{\sqrt{2}}{2}$.



$w = F\,d$
$= 500 \cdot \frac{\sqrt{2}}{2}\, 100$
$= 25000\sqrt{2}$

A) $100\sqrt{2}$
B) $5\sqrt{2}$
C) $\sqrt{2}$
D) 0
E) $25000\sqrt{2}$

---

3) Let u = (1, 2, -3) and v = (2, 4, -7). Which one is **true?**, where x is the cross product, . is the dot product. 3) ____

~~A) u and v are orthogonal vectors~~
~~B) u x v = (4, 1, - 1)~~
~~C) u . v = - 1~~
~~D) u x v = - v x u~~
E) u and v are independent vectors

$v = c \cdot u$

$u \cdot v = 2 + 8 + 21 = 31$

1 2 -3
2 4 -7

$u \times v = (-2, \ , \ )$

---

4) Let A be a 9 x 9 invertible matrix. Then Rank is

A) 9 B) 18 C) 81 D) 0 E) 99

$\|u\| = \sqrt{1+0+1} = \sqrt{2}$

5) Let u = (1, 0, 1) and a, b be positive numbers such that $\| au \| + \| bu \| = 2\sqrt{2}$. Then a + b is

$\underbrace{|a|}_{a}\|u\| + \underbrace{|b|}_{b}\|u\|$

$\boxed{a + b = 2}$

$\underbrace{\|u\|}_{\cancel{\sqrt{2}}}\left[a + b\right] = 2\cancel{\sqrt{2}}$

A) $\frac{1}{2}$ B) 1 C) 2 D) 3 E) 0

---

6) Let u, v, and w be some vectors in R^3. Which one of the following operations **does not make sense (not defined) ?**, where x is the cross product, . is the dot product 6)

A) w x (w x v) - (1, 2, 3)
B) $\|u + w - 2w\|$
C) (u . w) + v
D) w x (u x v)
E) (u . u) - $\|u + v\|$

$2 + (1, 3, 5) = ?$

---

7) Let u = (1, 2) and v = (0, 3) and let a = (2, -3), b = (3, 4) and c = (1, 0). Which one(s) of a, b, c can be written as a **linear combination** of u and v ?

A) Only a
B) None of a, b, c
C) a, b and c
D) Only c
E) Only b

$a = x \cdot u + y\,v$
$(2,3) = x\,(1,2) + y\,(0,3)$
$(2,3) = (x, 2x) + (0, 3y)$
$(2,3) = (x, 2x + 3y)$
$x = 2$
$2x + 3y = 3$
$4 + 3y = 3$
$3y = -1$
$y = -1/3$

Example

$u = (1,2)$, $v = (2,4)$. $a = (3,5)$ can be written as a linear combination of u and v ?

$a = x \cdot u + y \cdot v$
$(3,5) = x(1,2) + y(2,4)$

$\begin{cases} x + 2y = 3 \\ 2x + 4y = 5 \end{cases} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}$

$\boxed{NO}$

what about $a = (3, 6)$

$(3,6) = x(1,2) + y(2,4)$
$3 = x + 2y$
$6 = 2x + 4y$

$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{Yes}$

Rank = 6

8) Let A be a 7 x 7 and Nullity = 1. Which one is **false ?**

A) Determinant of A is zero. T
B) Ax = 0 has only trivial (zero) solution.
C) A is not invertible. T
D) Rank is 6. T
E) Row vectors are dependent. T

SHORT ANSWER.

= nullity

9) Find the number of parameters in the general solution of Ax = 0 if A is a 5 x 7 matrix of rank 3 ? 9) ___________

Rank = 3
$\boxed{\text{Nullity} = 4}$

$\|a\| = \sqrt{5}$

10) Let u = (-1, 3), a = (1, 2). Then proj_a u is equal to 10) _____

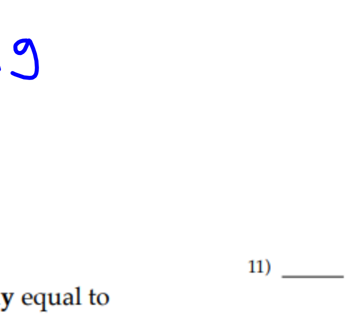
A) (-2, 2) B) (2, 0) C) (2, 4) D) (1, 2) E) (3, 1)

$\frac{u \cdot a}{\|a\|^2}\,a = \frac{5}{5}(1,2) =$

---

11) The area of the triangle with vertices A (0, 1, 4), B (- 5, -1, 2), C (6, 2, 1) is **approximately** equal to

$AB = (-5, -2, -2)$
$AC = (6, 1, -3)$
$AB \times AC = (8, -27, 7)$
A: Triangle $\frac{1}{2} \cdot \|AB \times AC\|$
$= \frac{1}{2}\sqrt{64 + (27)^2 + 49}$
$= 14.5$



11) The area of the triangle with vertices A (0, 1, 4), B (- 5, -1, 2), C (6, 2, 1) is **approximately** equal to 11) _____

A) 14.5 B) 13.5 C) 13 D) 12.5 E) 15.5

[1 2 3] Then Rank - Nullity is

12) Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. Then Rank - Nullity is 12) _____

3x3

A) - 1 B) 2 C) 3 D) 4 E) 0

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Rank = 1
Nullity = 2

$x_1, x_2 \geq 0$

5. Maximize

$$Z = 2x_1 + x_2$$

subject to

$$x_1 - x_2 \leq 1$$
$$5x_1 + 4x_2 \leq 20$$
$$x_1 + 2x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

6. Maximize

$x_1 - x_2 + S_1 = 1$

$5x_1 + 4x_2 + S_2 = 20$

$x_1 + 2x_2 + S_3 = 8$

$-2x_1 - x_2 + Z = 0$

departing / entering

	x_1	x_2	S_1	S_2	S_3	Z	R	
S_1	1	-1	1	0	0	0	1	1:1=1
S_2	5	4	0	1	0	0	20	20:5=4
S_3	1	2	0	0	1	0	8	8:1=8
Z	-2	-1	0	0	0	1	0	

entering / depar.

	x_1	x_2	S_1	S_2	S_3	Z	R	
x_1	1	-1	1	0	0	0	1	~~1:-1=-1~~
S_2	0	9	-5	1	0	0	15	15/9=1…
S_3	0	3	-1	0	1	0	7	7:3=2…
Z	0	-3	2	0	0	1	2	

$-\frac{15}{9}+2$

$\frac{15}{9}-1$

$-\frac{45}{9}+7=$

$\frac{63}{45}$

18

1	-1	1	0	0	0		1
0	1	-5/9	1/9	0	0		15/9
0	3	-1	0	1	0		7
0	-3	2	0	0	1		2

	x_1	x_2	S_1	S_2	S_3	Z	R
x_1	1	0	4/9	1/9	0	0	24/9
x_2	0	1	-5/9	1/9	0	0	15/9
S_3	0	0	6/9	-3/9	1	0	2
Z	0	0	1/3	1/3	0	1	7

max. $Z = 7$

$x_1 = 24/9$

$x_2 = 15/9$